Mathematics and Design Education

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Introduction

Many people believe that mathematical thought is an essential element of creativity. The origin of this idea in art dates back to Plato. Asserting that aesthetics is based on logical and mathematical rules, Plato had noticed that geometrical forms were "forms of beauty" in his late years. Unlike his contemporaries, he had stressed that the use of geometrical forms such as lines, circles, planes, and cubes in a composition would aid to form an aesthetics.[1] The rational forms of Plato and the rules of geometry are the basis of antique Greek art, sculpture, and architecture and have influenced art and design throughout history in varying degrees. This emphasis on geometry has continued in modern design, reflected prominently by Kandinsky's geometric classifications.[2]

Mathematics and especially geometry have found increasing application in the computer-based design environment of our day. The computer has become the central tool in the modern design environment, replacing the brush, the paints, the pens, and pencils of the artist. However, if the artist does not master the internal working of this new tool thoroughly, he can neither develop nor express his creativity. If the designer merely learns how to use a computer-based tool, he risks producing designs that appear to be created by a computer. From this perspective, many design schools have included computer courses which teach not only the use of application programs, but also programming to modify and create computer-based tools.

In the current academic educational structure, different techniques are used to show the interrelationship of design and programming to students. One of the best examples in this area is an application program that attempts to teach the programming logic to design students in a simple way. One of the earliest examples of such programs is the Topdown Programming Shell developed by Mitchell, Liggett, and Tan in 1988.[3] The Topdown system is an educational CAD tool for architectural applications, with which students program in Pascal to create architectural objects. Other such educational programs have appeared since then. A recent fine example is the book and program called "Design by Number" by John Maeda.[4] In that book, students learn programming by coding in a simple programming language to create various graphical primitives.

1 Reginald Hackforth, (English translation), *Plato's Examination of Pleasure, The Philebus* (Cambridge: The Cambridge University Press, 1945), 100.

2 Wassily Kandinsky, *Point, Line to Plane* (New York: Dover Publication Inc., 1979).

3 William Mitchell, "The Topdown System and Its Use in Teaching, an Exploration of Structured, Knowledge-based Design" in ACADIA'88 Workshop Proceedings: *Computing in Design Education,* ed. by P.J. Bancroft (Michigan: University of Michigan, 1988), 251-262.

4 John Maeda, *Design by Numbers* (Cambridge, MA: MIT Press, 1999).

However, visual programming is based largely on geometry, and one cannot master the use of computer-based tools without a through understanding of the mathematical principles involved. Therefore, in a model for design education, computed-based application and creativity classes should be supported by "mathematics for design" courses. The definition of such a course and its application in the multimedia design program is the subject of this article.

Mathematics Instruction in Design Education

Mathematics courses often are offered in the freshman year, as part of a common scientific core. However, in that case, the students do not consider the subject as a part of their creative development, and regard it as a boring compulsory course: a monotonous repetition of high school subjects. Our aim is the correct placement of mathematics in the context of design education, and the organization of the topics in the course from this viewpoint. In designing the course, we have taken into consideration the following points:

1. Our aim is to highlight the relationship of each topic with design and graphics. The students will place the topic in perspective as to where and in which stage of their design education it fits into.
2. This placement in perspective should be so successful that it will instill an enthusiasm for mathematics. The students should be able to draw parallels between the beauty in design and the mathematical forms that underlie and develop an appreciation for the beauty in mathematics.
3. Most important, the knowledge acquired in this course should enhance and open new horizons in creativity.

We have designed the course with these concerns, and have included topics which are relevant from the viewpoint of design and computer graphics. The subjects in the course are grouped not in the traditional way, but as a designer would need and use them. Each topic in the course is introduced after a discussion of why we need this topic. The discussion is supported by an analysis of specific examples from art and design history, where appropriate. For example, after a discussion of hyperbola, their use in the airport by Saarinen [5] is illustrated. Homework may be assigned to find similar examples from art and design history, and to contrast these examples. The topic is then discussed in detail with the underlying theory; with the specific application being worked out in the form of examples and homework assignments. As a last step, the student visualizes the graphical application of the mathematical concept through the use of an educational program developed for this course in the computer laboratory.

5 Kenneth Frampton, *Modern Architecture: A Critical History* (London: Thames and Hudson, 1992).

Breakdown of Topics

We have selected the topics covered in this course from the standpoint of their utility in design and graphics. Table I summarizes these topics. As we have outlined above, in the implementation of the course, we introduce each topic by a discussion of where one might need it. We follow this discussion by an example from art and design history. We analyze this example in depth; and give the students an assignment to research this topic in depth. We then undertake the discussion of the mathematical subject in depth. We select classroom examples and homework assignments from graphical applications. Programming assignments in related topics are assigned in a concurrent course in programming. In this course, our aim is to make use of a visualization tool that has been developed for the course. The laboratory session enables the students to visualize the mathematical forms, and to experiment with the mathematical form in creating new designs using this visualization tool. Here, we discuss in detail how each topic is developed and discussed and what the visualization laboratory module for each topic aims to illustrate.

Vectors in two-dimensional space: The introduction to the course starts with a discussion session: What is the simplest form of all? We show that all forms may be represented as a collection of points, the simplest form of all. Designs involving points, such as the work by Müller-Brockmann (figure 1), are shown. The mathematical counterpart, a vector in two-dimensional (2D) space is introduced. The discussion of vectors continues with norms of vectors, inner products, and angles and leads to different ways of representing lines. We illustrate these concepts with classroom examples of drawing lines with the desired slopes and specific angles. The visualization laboratory module for this topic enables students to experiment with placing points and drawing lines on the screen. Unlike a conventional application program in which the mouse is used, the students are asked to enter x and y coordinates of points, and the slopes and intercepts of lines. Different options enable the students to experiment with random numbers and special functions to specify these coordinates; encouraging them to create different designs using mathematical formulas.

Polygons: We discuss the polygon as a graphical primitive, with emphasis on modeling real-world objects. We study polygonal models of real world objects as classroom examples. The discussion of lines already has prepared the basis for polygons. Here, the discussions continue with special polygons: Squares, rectangles, parallelograms, triangles. Special triangles are discussed, and the angles between lines are revisited. The definitions of area, perimeter, and convexity are given. The visualization laboratory module enables students to experiment with drawing polygons with specific

Table I
Course Plan of Mathematics for Design

Traditional Mathematical Theme	**Relationship to Graphics**	**Examples from Art and Design**	**Computer Lab Applications**
Vectors in 2D space; their norms; vector addition and subtraction dot products; explicit, implicit and parametric representations for a line	Points in 2D Lines and Line segments	Paintings by Seurat	Visualization of points, lines and line segments; selection and placement of these primitives in a composition
Polygons; convexity; special polygons	triangles squares other polygons	Pied Mondrian Red, Yellow, Green	Visualization of polygons and compositions involving different polygons
Quadratic curves; their implicit and parametric forms; tangents to curves	circles ellipses hyperbola parabola	Saarinen's TWA Building[5]	Visualization of these primitives, effect of various parameters
Parametric polynomial curves; Bezier curves; tangents to Bezier curves	Freeform curves	Malevich sketches, industrial design at Renault	Visualization of Bezier curves, effect of changing control points, joining different curve segments
Boolean operations	Boolean AND, OR and set difference	Spreckelsen and Andreu's Arche de La Defense[5]	Using Boolean operations on Polygons to create compositions involving different colors
Matrices; matrix products, inverses ; homogeneous coordinates	transformations rotation translation scaling symmetry	Mondrian and Malevich;	Application of these transformations to the primitives developed: concatenation of different transformations
Perspective transformation	Points and lines in 3D Perspective transformation	Perspective examples from Renaissance Art	Visualization of 3D points and lines, development of perspective
Vectors in 3D space; Planes; normals, vector cross products	Planes in 3D; polygons in 3D	Wright's Falling Water[5]	Visualization of polygons in 3D. Specifying planes in different ways
Polyhedra Booelan operations revisited	Cubes, pyramids, etc	Louvre's Pyramid[5]	3D visualization of polyhedra
Quadratic surfaces	Sphere Ellipsoid Torus	Etienne Boullee's Newton Monument[6]	Visualization of spheres, ellipsoids, toruses, specification and effect of parameters
Matrices revisited	transformations in 3D	Architectural examples from Eisenman[5]	Rotation, and scale about an axis; concatenation of transforms in 3D
Fractals	Texture		Mapping texture to primitives
Angles between 3D vectors revisited	Illumination shadows illumination models	Francis Coppola's Drakula, Rumble Fish	Effect of placing light sources in 3D space, effects of angles between surfaces and light
Interpolation	Animation	Luxo Jr. by Pixar[7]	Specifying interpolating curves for animation

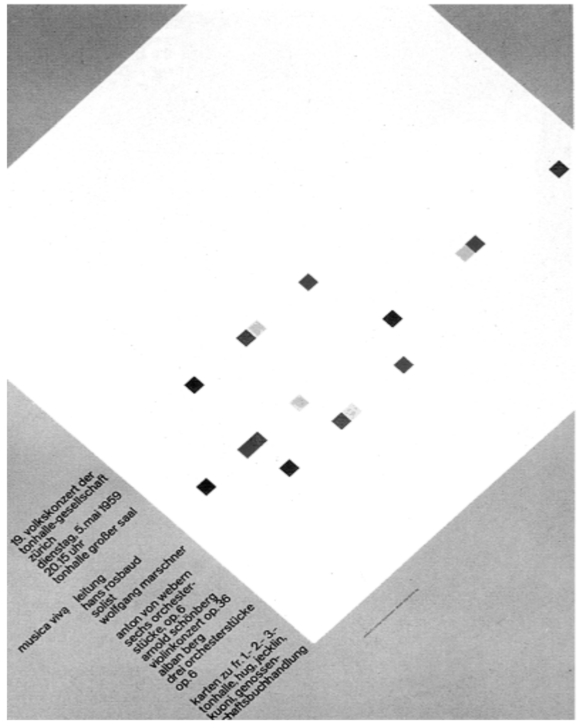

Figure 1
A design involving points and lines, titled *Musica Viva* by Müller-Brockmann. © Copyright, Poleittris 1959 8033 Zürich. From W. N. Meggs and E.P. Meggs, *A History of Graphic Design*, 330 (1998.) Reprinted by permission of John Wiley & Sons, Inc.

angles. The students can draw polygons by specifying coordinates of vertices or by specifying angles between edges and length of edges. As an option, the student can specify these parameters in terms of random numbers and special functions, and create designs involving color-filled polygons.

Quadratic curves: Discussion on modeling of circular objects leads into quadratic curves. Implicit and parametric representations of circles, ellipses, parabolas, and hyperbolas are discussed. Discussion continues with differentiation of quadratic curves and tangent lines. A possible classroom example is finding the intersection of a line and a circle. The visualization laboratory module enables students to specify parameters such as centers, radii, etc., to draw quadratic curves.

Parametric Polynomial curves: The discussion starts with the history of computer-aided design and of Bezier's work at Renault. The representation of freeform curves in terms of cubic Bezier polynomials is discussed. Control points and joining of different curve segments is shown with examples. Visualization laboratory experiments let the students model 2D objects with the selected number of Bezier curve segments.

Boolean Operations: More complex objects may be defined by combinations of simple primitives. Boolean operations are introduced for this purpose. Two-dimensional image objects are represented as sets, and union and intersection are defined on these sets. Regularized Booelan operations are defined. The visualization laboratory module enables students to create objects using unions and intersections of closed objects such as polygons and circles.

Matrices: Copying, moving, enlarging and rotating objects: These simple transformations are an important part of any drawing program. They can be represented in terms of matrices. Starting from this point, we introduce matrices. Rotation and scale operations are represented as matrix operations. Homogeneous coordinates are introduced and translation operation is represented in matrix form. Concatenation of transformations and matrix multiplication is discussed. Examples are studied to show that matrix multiplication is not commutative. Visualization laboratory examples direct the student to specify parameters of transformation matrices, and to specify the order of transformations to achieve the desired object placement.

Perspective transformation: The simplest two primitives studied, points and line segments, are extended to three dimensions (3D). We illustrate the effect of perspective with examples, which show that "further objects look smaller." Perspective transformation is formulated in matrix form. Visualization laboratory experiments enable students to change the position of viewpoint and position of objects, and to see the resulting change in perspective projections of objects.

Vectors in three-dimensional space: We extend all the primitives studied to three dimensions. Planes in 3D space are introduced and different ways of specifying a plane are discussed. Cross products of 3D vectors are introduced and plane normals are defined. Nonplanar polygons are illustrated. The visualization laboratory module enables students to visualize planes specified by different methods: by supplying parameters to the plane equation, by specifying three points on a plane, and by specifying a point and a normal vector.

Polyhedra: The discussion of polyhedra starts with one of the most famous polyhedral forms: the pyramid. Starting from the pyramid and the cube, we introduce different polyhedra, and state conditions for convexity and simplicity. We show the topological atlas of polyhedra, and define the corresponding data structure. In the visualization lab, students learn to use this data structure to define simple polyhedra. Booelan operations enable the students to define more complex forms.

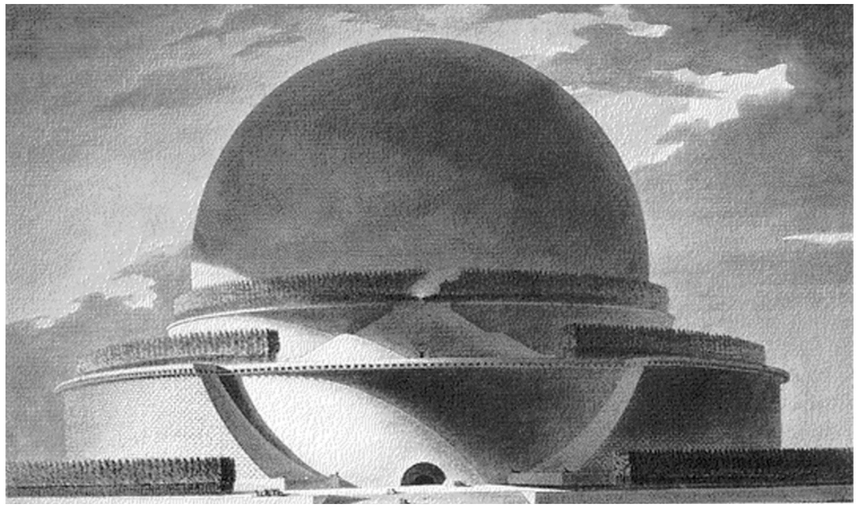

Figure 2
Etienne Boullee's design of a cenotaph for Newton.[6]

Quadratic surfaces: Extensions of quadratic curves to 3D yields quadratic sur-faces such as spheres, ellipsoids, and toruses. We discuss the use of these surfaces in architecture (figure 2). The implicit and parametric forms of these surfaces are introduced. In the visualization laboratory, students experiment with parameters to create different forms, and use intersections with each other and with planes to visualize cross sections.

Matrices revisited: Transformations defined on 2D are extended to 3D. We define the rotations about coordinate axes and about an arbitrary direction. We illustrate scale along an arbitrary line. In the visualization laboratory, students see the effects of rotations and scale about an arbitrary direction, as well as the effects of transformation order.

Fractals: Discussion starts with examples from nature: the texture of leaves, tree branches and microorganisms. Recursive and procedural definitions that mimic this self-similar growth patterns are introduced. Different fractals are discussed. In the visualization laboratory, students create different fractals by supplying rules.

Angles between vectors revisited: We discuss the behavior of light and illumination. We introduce simple illumination models as applications of vector operations. Students apply these illumination models in the visualization laboratory and see the effect of angles between surface normals, lighting, and viewing directions.

Interpolation: Animation by key-framing is the concept used to illustrate interpolation. We discuss a well-known early example of computer animation by key framing, *Luxo Jr.,* directed by Lassater,[7] as an example (figure 3). Linear, quadratic, and B-spline interpolation is discussed. Selection of keyframes and the interpola-

6 Sketch of a Design for a Cenotaph for Sir Isaac Newton, Bibliotheque Nationale, Paris, 1784) A.M. Vogt, *Art in the Nineteenth Century*, 9-10.

7 John Lasseter, "Principles of Traditional Animation Applied to Three-dimensional Computer Animation," *Computer Graphics* 21:4 (1987): 35-44.

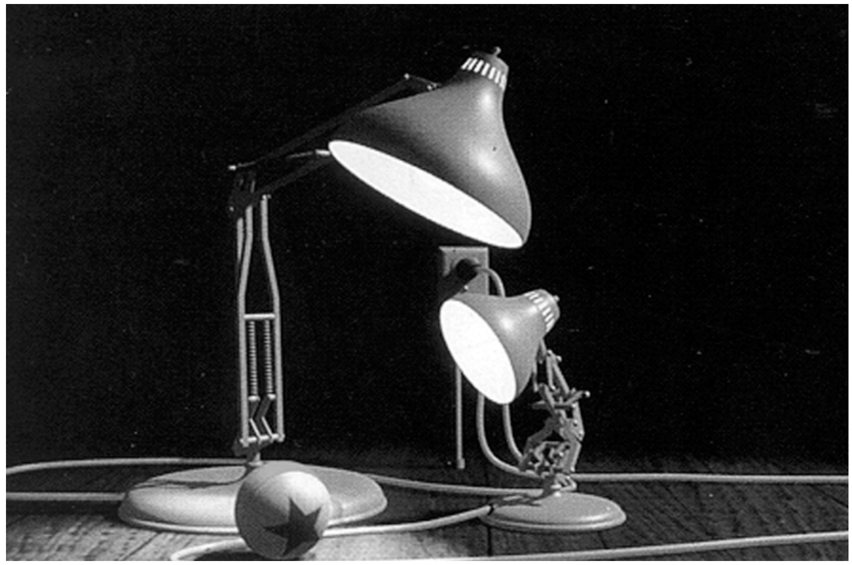

Figure 3
An example of keyframe animation: *Luxo Jr* Directed by John Lasseter. (© Pixar Animation Studios, 1986).

tion function is discussed with examples. In the visualization laboratory, students get the chance to animate simple objects by specifying key frames and interpolation functions.

Conclusion

The purpose of the course structure described above is to teach the student the mathematical structure behind "form." We explain the relationship between this mathematical structure and the design components through important examples in the history of art and design, and reinforce this relationship by explaining the underlying mathematical structure behind these examples. The visualization of the mathematical concepts are achieved through a visualization laboratory. A dedicated visualization tool is being developed for this course.

Furthermore, as the related topics are covered, students are reminded of Kandinsky's geometric classifications covered earlier in history of art, basic design, and drawing courses. In that way, students will perceive the mathematical concepts covered in this course not as independent, isolated topics, but as an integral part of all the design education they get; with reference to all of the other courses and topics.

Some difficulties in teaching the course involve the resources: While an instructor with a computer graphics background can teach the course, a second instructor with an art and design history background is needed to give insight into the examples discussed. The course is being offered the second time this year. In order to evaluate the effectiveness of our methods and the degree to which we reach our goals, we plan to do follow-up studies when these students graduate.

We expect that this new way of teaching mathematics to design students, making them analyze the role of mathematics in design applications, will make the course more fun and that the mathematical concepts learned in the course will enhance their creativity.